



Angle Determination in ECG Using Trigonometric Formula

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Abstract

Angle determination in ECG can be calculated using trigonometric mathematical formula. Conventionally, they used to analyze the axis (angle) of P, QRS and T wave by the 4 quadrant method using the hex-axial reference system. But the medical staffs have to be very thorough with the formulation of hex-axial reference system before efficient interpretation of the axis of ECG waves. The angle (axis) can be easily calculated using trigonometric formula by scientific calculator which may help the junior medical staffs to interpret the axis quickly and easily.

Keywords: ECG Wave; Axis determination; Tan α

Introduction

The hex-axial reference system is formulated using the bipolar limb leads (LI, LII & LIII) and unipolar limb leads(aVR, aVL & aVF). The limb leads LI(0°), LII(60°), LIII(120°), aVR(-150°), aVL(-30°) and aVF (90°) are forming two equilateral triangles in the hex-axial reference system with heart in zero potential situated at the centre of the hex-axial reference system [1-4]. The normal QRS (Vector) axis is from -30° to 90° and the normal T wave axis is between 0° and 90° . The T wave axis is analyzed in its relationship to the QRS axis and the normal QRS-T angle does not usually exceed 60° in the frontal plane in the hex-axial reference system [1,5,6].

The angle determination in ECG has a significant role in the diagnosis of various cardiac conditions like intra-ventricular conduction defect, bundle branch block, hypertrophy, ischemia, injury, infarction etc. [1,5,6]. This article summarizes the derivation, application and the clinical significance of using the corrected trigonometric formula to calculate the angle or axis of cardiac waves in ECG interpretation.

Materials and Methods

Angle Determination in ECG

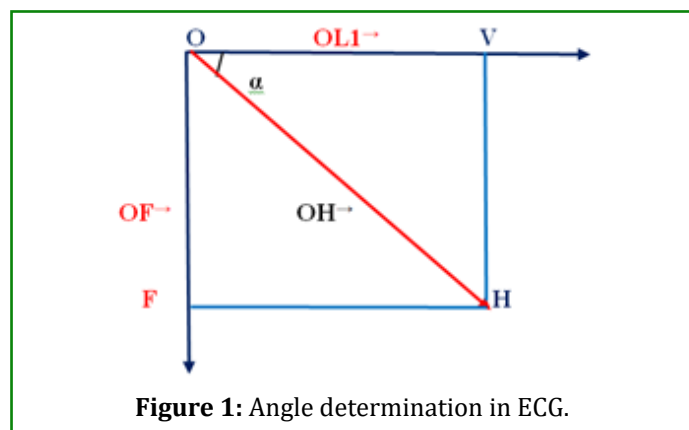


Figure 1: Angle determination in ECG.

- **O:** point of origin (zero at the centre of the hex-axial reference system)
- **OL1→:** lead I Vector (Unit vector in metre)
- **OV:** Voltage recorded in lead I

- **OF**→: aVF lead Vector (Unit vector in metre)
- **OF**: Voltage recorded in aVF lead
- **OH**→: Heart Vector (Volt/metre)

From the tip of the cardiac vector, two perpendicular lines are drawn which meet the lead I at point V and lead aVF at point F.

In the right angled triangle OVH:

$\tan \alpha = \text{opposite side}/\text{adjacent side}$

$\tan \alpha = \text{VH}/\text{OV}$

$\tan \alpha = \text{OF}/\text{OV}$ (from the diagram it is very clear that $\text{OF} = \text{VH}$)

$$\tan \alpha = \text{aVF}/\text{lead I}$$

The unipolar and bipolar limb leads have different resistance. So, the correction factor of 1.154 is to be applied [1-4,7]. The formulae to calculate the angle determination in ECG is as follows.

$$\tan \alpha = (1.154 * \text{aVF}) / \text{Lead I}$$

The values of tangent (tan) angles for the following angles are given in Table 1.

Tan angles	$\tan 0^\circ$	$\tan 30^\circ$	$\tan 45^\circ$	$\tan 60^\circ$	$\tan 90^\circ$
Values	0	$1/\sqrt{3}(0.577)$	1	$\sqrt{3}(1.732)$	Infinity(undefined)

Table 1: Tan angles Values.

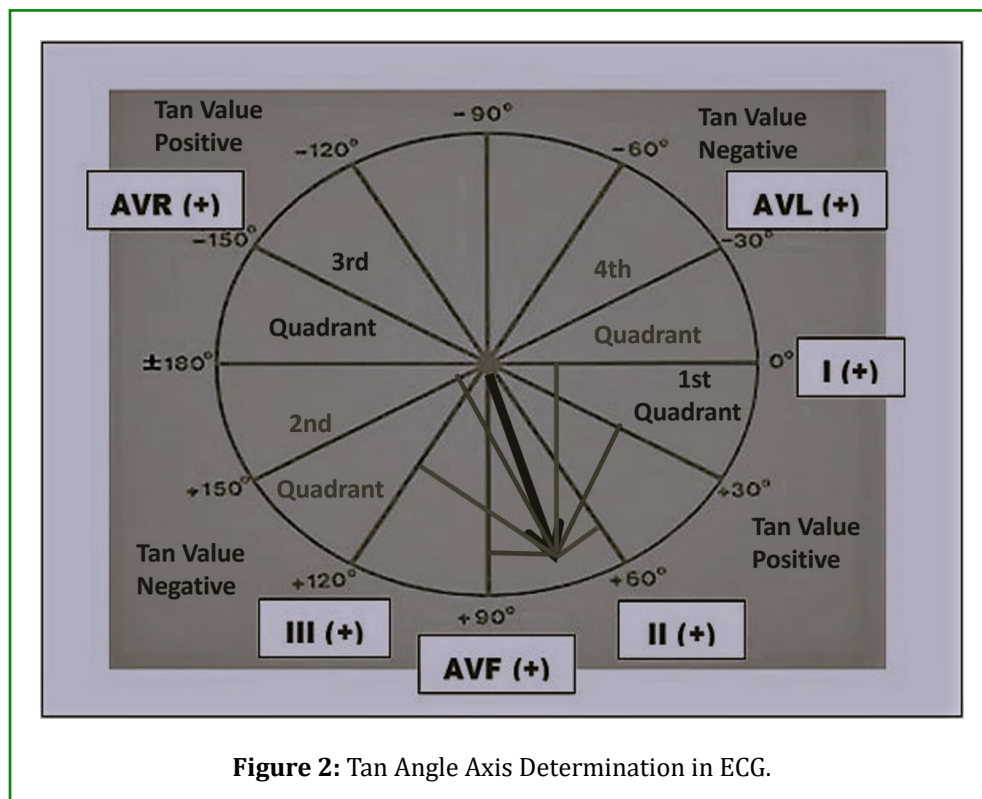


Figure 2: Tan Angle Axis Determination in ECG.

Angle determination using 4 quadrant of the hexaxial reference system is clearly depicted in the Figures 1 & 2 [1]. The tan value is negative in both the 2nd and 4th quadrant. For 2nd quadrant in the hex-axial reference system (lead I negative & aVF positive), angle should be subtracted from 180 degree. For the 4th quadrant (lead I positive & aVF negative) angle should be subtracted from 360 or zero degree [1,2]. The relationship between bipolar and unipolar limb leads are

related using well known equations [1-4,7].

Results

Angle determination was done using the trigonometric formula. 5 examples are cited in Table 2. The relationship of the voltages of the various limb leads were utilized to derive the voltages measured in each leads which are shown in mm.

Leads	Voltages measured in mm in each lead is taken				
	1st example	2nd example	3rd example	4th example	5th example
Lead I	3	-0.2	2	9	-1.25
Lead II	2	0.8	3	-3	-1
Lead III	-1	1	1	-12	0.25
aVR	-2.5	-0.3	-2.5	-3	1.125
aVL	2	-0.6	0.5	10.5	-0.75
aVF	0.5	0.9	2	-7.5	-0.375
Angle	11 ⁰	100.9 ⁰	49 ⁰	-43.88 ⁰	-160.91 ⁰
Location in Hex-axial Reference system	1 st Quadrant	2 nd Quadrant	1 st Quadrant	4 th Quadrant	3 rd Quadrant

Angle determination is done using the formula: $\tan \alpha = (1.154 * aVF) / \text{Lead 1}$

Table 2: Angle calculation using voltages in mm in each limb leads of ECG.

Discussion

In the conventional approach using the 4 quadrant method in hex-axial reference system, the angle determination in ECG is done as follows. If the cardiac vector is perpendicular to a particular lead, then the net voltage recorded in that particular lead will be zero. If a cardiac vector is parallel to a particular lead, then the net voltage recorded will be maximum in that particular lead [1,2,5]. Using these concepts, either equiphasic (or net zero voltage) or maximum voltage recorded in a particular lead (3 bipolar and 3 unipolar limb lead) is checked in the electrocardiogram. Then the angle or axis is analyzed using the hex-axial reference system using the orientation of various leads with the resultant cardiac vector. For example, if the cardiac vector is perpendicular to aVF or parallel to Lead I, then the angle will be zero degree. If the cardiac vector is parallel to aVF or perpendicular to Lead I, then the angle will be 90⁰.

Using the hex-axial reference system, the angle can be determined but it is difficult to calculate all angles and also it requires thorough knowledge of the various leads oriented in the hex-axial reference system. But the application of trigonometric formula helps us in better understanding of the axis calculation. Suppose if the value of aVF is zero, the corresponding angle for that tan value is zero degree. If the value of the Lead I is zero, the corresponding angle for that tan value ($1/0 = \text{infinity}$) is 90⁰. If the cardiac vector is not parallel and not perpendicular to any of the limb leads, then it will be very difficult to determine using the quadrant method using hex-axial references system and only experts can analyze and arrive it. But using the trigonometric formulae,

it is easy to calculate any angle accurately and quickly which are clearly cited with examples in table 2.

Conclusion

Heart disease remains a major public health problem. ECG is one of the most important diagnostic modality for the diagnosis of various cardiac conditions. So accurate, easier and quicker angle determination in ECG by medical staffs can help in identification of various cardiac problems thereby resulting in reducing the mortality and morbidity of this number one killer disease of the world.

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